A NEW TECHNIQUE FOR CALIBRATING DUAL SIX-PORT NETWORKS WITH APPLICATION TO S PARAMETER MEASUREMENTS

Leon Susman
Sperry Research Center
100 North Road
Sudbury, Massachusetts 01776

ABSTRACT

This paper describes a new technique for the simultaneous calibration of two six-port networks as reflectometers. Once calibrated, the complete S parameters of an unknown two-port network can be obtained from power measurements alone. The technique requires only a uniform transmission line, of unknown length and loss, and a short circuit as calibrating standards. Its application to an automated microwave measurement system is described and potential error sources are quantitatively discussed.

Introduction

The basic concepts involved in six-port measurement systems represents a drastic departure from the conventional microwave measurement art. Rather than relying on the quality and precision of microwave components (such as high directivity and low VSWR in directional couplers) needed for accurate measurement systems, the six-port concept attempts to quantitatively account for these imperfections by measurements made during the so-called calibration phase. In this respect it is similar in philosophy to current automatic network analyzer techniques. However, inherent in the six-port concept is the fact that both the calibration of the measuring network and the subsequent measurement of some unknown two-port requires only power measurements. This greatly simplifies the task of automating the measurement technique, since no frequency conversions is required, and a wide variety of power measuring devices are available to interface with a digital system. Six-port measurement techniques have thus been under investigation for several years as a means of reducing the cost of automated microwave measurements without sacrificing accuracy.[1,2]

This paper describes a self calibration technique for the simultaneous calibration of two six-port networks. The only standards used are a uniform transmission line of unknown length and loss, and a fixed short circuit. After calibration each six-port becomes a reflectometer and with a two-port network placed between them the S parameters of the two-port can be computed from power measurements at the available output ports of the two six-port networks.

The procedure outlined here represents a substantial improvement over the calibration technique previously proposed in several important respects.[3] On a conceptual level we show that the calibration constants can be obtained as the eigenvectors of a certain eigenvalue problem which is established by the particular calibrating procedure. This allows the use of linear algebra algorithms which clarifies the theoretical basis for the procedure. It is further shown that the associated eigenvalues are directly related to the physical parameters of the uniform transmission line standard. As long as the loss and phase shift through the transmission line is stable, their actual value need not be known. Unlike previous calibration techniques there is no requirement for constant incident power and thus no need for leveling loops. Finally the technique is usable for a wider variety of connector types since sexless connectors are not required.

A computer simulation of this calibration technique has been written and it has been confirmed that the calibration procedure and the subsequent accuracy is, ideally, independent of the quality of components used in the six-port networks. On the other hand the finite resolution capability of any power detector limits the S parameter measurement accuracy. The quantitative tradeoff between desired accuracy and required digital resolution in the power detectors is presented.

Calibration Procedure

Consider the network configuration shown in Fig. 1. It consists of two six-port networks excited by a signal from a common source. Each six-port has four power detectors attached to their respective outputs.

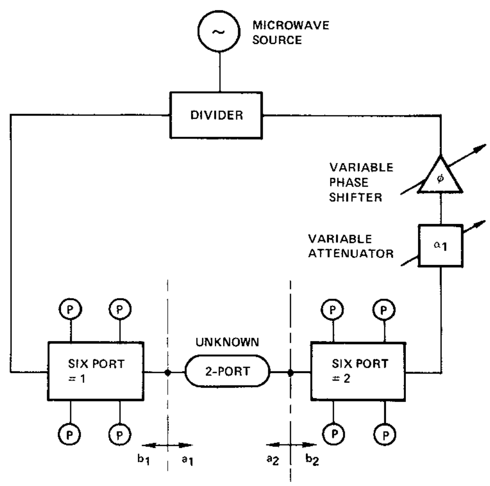

Fig. 1 Dual Six-Port Block Diagram

The variable attenuator and phase shifter are used to vary the complex ratio of incident signals on the two networks. For each six-port network we can write a linear matrix relationship between the power readings and certain quadratic functions associated with the scattering variables of the six-port. In particular the following matrix equations are valid for the two networks

$$\vec{a}_{q1} = C_1\vec{P}_1 \quad ; \quad \vec{a}_{q2} = C_2\vec{P}_2 \qquad (1)$$

where

$$\vec{a}_{q1} = \begin{bmatrix} |a_1|^2 \\ a_1 b_1^* \\ a_1^* b_1 \\ |b_1|^2 \end{bmatrix} \; ; \quad \vec{a}_{q2} = \begin{bmatrix} |a_2|^2 \\ a_2 b_2^* \\ a_2^* b_2 \\ |b_2|^2 \end{bmatrix} . \qquad (2)$$

Matrices C_1, C_2 are 4 x 4 complex matrices whose entries are the calibration constants for network 1 and 2 respectively. The column matrices $\vec{p}_1$, $\vec{p}_2$ contain the power readings from network 1 and 2 respectively. Equation (1) is the basic equation between the observable power readings and the input quantities defined by (2).

By placing a two-port network between the two six-ports we can constrain the vectors $\vec{a}_{q1}$ and $\vec{a}_{q2}$ to satisfy an equation of the form $\vec{a}_{q1} = T_q \vec{a}_{q2}$, where the entries of T_q are related to the transfer matrix of the two-port.[1]

The calibration procedure consists of three steps. First the two six-ports are connected together. An excitation is applied to both networks, resulting in power readings vector $\vec{p}_1$ and $\vec{p}_2$. This is repeated for a total of at least 4 ratios of incident signals (obtained by varying the attenuator and phase shifter). This sequence of measurements are summarized by two equations similar to (1).

$$A_1 = C_1 P_1 \; ; \quad A_2 = C_2 P_2 \; ; \quad A_1 = T_q A_2 \qquad (3a)$$

where A_1, A_2, P_1, P_2 are 4 x 4 matrices whose columns are the vectors defined in equation (2). For this direct connection T_q takes the simple skew diagonal form

$$T_q = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} \qquad (3b)$$

The calibration continues by replacing the direct connection with a uniform length of transmission line. The measurements are repeated and lead to an equation similar to (3a) with different power matrices P_1', P_2' and a new T_q matrix.

$$A_1' = C_1 P_1' \; ; \quad A_2' = C_2 P_2' \; ; \quad A_1' = T_q' A_2' \qquad (4a)$$

with T_q' now given by

$$T_q' = \begin{bmatrix} 0 & 0 & 0 & e^{-2\alpha\ell} \\ 0 & 0 & e^{+2j\beta\ell} & 0 \\ 0 & e^{-2j\beta\ell} & 0 & 0 \\ e^{+2\alpha\ell} & 0 & 0 & 0 \end{bmatrix} \qquad (4b)$$

The transmission line is assumed to have length ℓ and propagation constant $\gamma = -\alpha + j\beta$. Equations (3) and (4) can now be solved for either calibration matrix C_1 or C_2. Specifically by eliminating C_2 the following similarity transformation is derived

$$C_1 \left[P_1' (P_2')^{-1} P_2 P_1^{-1} \right] C_1^{-1} = T_q' T_q \qquad (5)$$

where C^{-1} denotes the inverse of C and the product $T_q' T_q$ is a diagonal matrix. Equation (5) shows that the calibration constants (the rows of C_1) are the eigenvectors of $\left[P_1' (P_2')^{-1} P_2 P_1^{-1} \right]$ (which are derived from the power readings). The eigenvalues are the diagonal entries of $T_q T_q'$. Since eigenvalues are only determined up to a multiplicative constant, the matrix C_1 is thus far determined up to a multiplicative diagonal matrix. Finally to determine this diagonal matrix requires one measurement of the output power meters with a short circuit terminating each six-port.

Measurement Procedure

Once the calibration matrix for the six-port networks are found each six-port becomes a calibrated reflectometer. It is well established that by placing an unknown two-port between two calibrated reflectometers the S parameters of the two port can be found. One technique for accomplishing this has been devised in the literature.[3] An alternate approach is provided by the techniques developed in this paper.

With an unknown two-port connected between the two reflectometers an equation similar to (4) can be written

$$C_1 P_1'' = T_q'' C_2 P_2'' \qquad (6)$$

This again requires 4 sets of measurements. Since the calibration matrices are known we can solve for the unknown T_q'' matrix. The S parameters of the two port are embedded in the computed entries of T_q'' but these can be easily de-embedded.[4]

Discussion

The calibration procedure can be generalized in several useful respects. We have assumed that only 4 sets of independent excitations are used during the calibration process. This leads to a completely deterministic set of equations involving only square matrices. Alternately additional attenuator-phase-shifter settings can be used to provide some redundant equations which can be solved in a least square sense. This process would tend to smooth out the measured data and improve the overall accuracy.

Several assumptions concerning the network model are hidden in the maze of matrix manipulation presented above. Throughout there is the assumption of single mode operation in the microwave network. In addition equations (3b and 4b), for the T_q and T_q' matrix, assumes that the same real port normalization has been chosen for the direct connection and the "standard uniform line". In this respect the uniform line could be replaced by a symmetric lossy two-port whose image impedance is known. The presence of some loss improves the convergence properties of the eigenvector routine since the presence of a double root is eliminated from the skew diagonal matrix in equation (4b).

Finally the short circuits used in the

calibration procedure, not only establish a scale factor for the calibration matrix, but also establishes the reference planes for the computed two-port S parameters.

Computer Simulation Results

Our purposes of performing a computer simulation of the six-port calibration and measurement technique are threefold. First there is the basic need to verify the calibration procedure. Secondly there is always, the concern that the sensitivity of the technique to small perturbations in the ideal model might be high, suggesting that a hardware implementation would be difficult, if not impossible. Finally the simulation program becomes the primary program from which the final operating version is generated. This allows debugging of the computer program to be performed on precisely controlled data rather than uncertain measurements. From these simulations, we have confirmed that all the procedures developed here are theoretically correct under ideal conditions. To examine the influence of nonideal behavior in the measurement process, we briefly explored several effects. These include generator mismatch and truncation error due to the limited resolution in the dual six-port power meters.

The results of a simulation for the dual six-port are shown in Figure 2. Theoretically the calibration procedure is independent of network mismatch, a fact which has been confirmed for a wide range of generator mismatch. Figure 2 shows the influence of limited digital voltmeter (DVM) resolution on the measurement accuracy of $|S_{12}|$ for various matched attenuators.

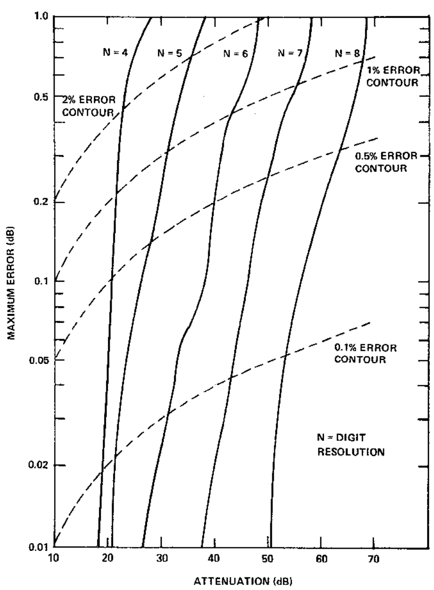

Figure 2 Maximum Error Due to Truncation of Power Readings to N Digits

The generator is assumed to be matched. As the number of digits being processed is increased, the accuracy increases sharply. It should be noted that in our experimental situation, the DVM usually produces a 4 digit number, which can be averaged to give at most a 5 digit number. According to Fig. 2, we would expect an error of 0.4 dB for a 40 dB attenuator with 5 digit resolution. Furthermore the curves of maximum truncation error are rather steep so that the error for a 50 dB attenuator is greater than 1 dB. We note that as the resolution capability improves larger attenuator values can be accurately measured. For all curves shown the corresponding phase error is much less sensitive to truncation and is consistently less than .5° and .1° for N = 3 and 4 respectively. Figure 2 graphically displays the influence of DVM resolution to dynamic range. It points out the necessity of increasing resolution to achieve larger dynamic range in the overall measurement system.

Acknowledgment

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